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ABSTRACT

The goal of this study was to develop a framework for classifying algebra story problems and to determine observed frequencies for each problem type. One thousand ninety-seven algebra story problems were selected from nine standard algebra textbooks. These are divided into eight families based on the nature of the source formula involved: for example, nearly 300 problems were classified in the "time rate family" because they were based on the source formula, "distance = rate x time" or "output = rate x time." Each family was divided into problem categories based on the general form of the story line: for example, the time rate family consisted of "motion," "current," and "work" categories. Each category was divided into templates based on the specific propositional structure of the problem: for example, there were a dozen templates for motion problems such as "overtake," "closure," "round trip," etc. This paper describes the procedure for generating families, categories, and templates and provides frequency counts for each observed template. Implications for fostering productive research and instruction are discussed. (Author)

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SERIES IN LEARNING AND COGNITION

Schemas for Algebra Story Problems

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Report No. 80-3

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Abstract

The goal of this study was to develop a framework for classifying algebra story problems and to determine observed frequencies for each problem type. 1097 algebra story problems were selected from nine standard algebra textbooks. These are divided into eight families based on the nature of the source formula involved; for example, nearly 300 problems were classified in the "time rate family" because they were based on the source formula, "distance = rate x time" or "output = rate x time." Each family was divided into problem categories based on the general form of the story line; for example, the time rate family consisted of "motion," "current," and "work" categories. Each category was divided ~~into~~ templates based on the specific propositional structure of the problem; for example, there were a dozen templates for motion problems such as "overtake," "closure," "round trip," etc. This paper describes the procedure for generating families, categories, and templates and provides frequency counts for each observed template. Implications for fostering productive research and instruction are discussed.

There has been growing interest in the study of how people solve algebra story problems--what Hinsely, Hayes and Simon (1977) call "those 20 Century fables." However, research in this area has been hindered by a lack of framework for describing differences and similarities among algebra story problems. For example, the same names such as "DRT" or "work" are used to characterize different problems by different investigators, while very similar problems may be given different names. Hence, it is difficult to generalize or compare from one study to another. While a review of the literature is beyond the scope of this paper (see Mayer, 1980), the goal of this paper is to provide a framework for describing types of algebra story problems and to provide norms for observed frequencies in algebra textbooks for each type.

Classificiation of Algebra Problems

There are two major schemes for classifying algebra problems: by the form of the underlying algebraic equations, or by the general form of the story line.

The form of the underlying equation refers to the structure of the solution equation. For example, a problem may require one equation with one unknown and two givens (such as a simple time-rate-distance problem); a problem may require solving simultaneous equations (such as in many age problems); a problem may involve a simple linear relationship between two variables (such as in direct variation problems); a problem may involve quadratics (such as area problems); and so on. Most algebra textbooks are organized into chapters based on the form of the underlying equation.

The form of the story line refers to major "categories" of problems such as motion, work, river, current, age, coin, etc. Hinsely, Hayes & Simon (1977) refer to these as "schemas" and suggest that there are at least 18 of them: triangle, DRT, averages, scale conversion, ratio, interest, area, max-min, mixture, river current, probability, number, work, navigation, progressions, progressions-2, physics, exponentials (see pages 93-4). Many textbooks explicitly name the major problem categories and provide solution procedures for each.

The two systems are not mutually exclusive. In general, certain major categories of problems (based on story line) involve characteristic underlying equations, as noted above. However, while categorizing problems into general groups, such as Hinsley et al., is a useful first step, there may be important levels both above and below the "category" level. Since these levels may be relevant to the way in which students work with problems, this paper will focus on the following levels of analysis.

problem format--What is a story problem? Can problems be divided into story problems and non-story problems? For non-story problems, can problems be further divided into equation, formula, number, and arithmetic word?

problem family and source formula--Many story problems have an underlying source formula; for example, "distance = rate x time" or "output = rate x time" are source formulas for the "time rate family." What is the nature of the underlying source formula for each problem?

problem category--This is the level of analysis that is usally presented in textbooks. Based on story line, for example, the "time rate family" can be broken into "motion," "current," and "work" categories. What is the category of the problem?

problem template, variation and modification--A more detailed level of analysis focuses on the propositional structure of the

> problem. For example, there are at least 12 different templates for motion problems, such as "overtake," "round trip," "closure." In addition, there are variations and modifications that may be introduced for each template. What is the template of the problem?

As can be seen, although the category level is the standard level of analysis, problems can be analyzed at a higher level (such as "family" or "source formula") and at more detailed levels (such as "template"). By investigating a large data base of story problems, this study will provide information concerning source formulas and templates in common use.

Selection of Data Base

Nine standard algebra textbooks currently approved for use in California secondary schools were selected, based on consultation with curriculum experts.[1] (A list of the books is given in the reference section of this paper). Then, a photocopy was made of each page that contained one or more algebra story problems. Finally, each problem was cut out from the photocopy and glued onto a 4 x 6 inch index card. There was one problem per card, with the page and book source written on the back.

A problem was included in the data base if it met the following criteria: (1) The problem was given as an exercise or a chapter test rather than as a worked out example in the body of the text. Thus, the data base contained problems that textbook writers assumed students should be able to solve. (2) The problem asked for a numerical answer rather than translating a story into equations, making a judgment about what actions should be taken, estimating an answer, or the like. (3) The problem used words rather than equations.

For example, Use R = D/T to find the value of R if D = 300 and T = 10" is not a story problem; "If a car travels 300 miles in 10 hours, what is its average speed?" is a story problem. (4) The problem had a story line consisting of characters, objects, and/or actions. For example, "Find five consecutive integers whose sum is 45" has no story line; the following problems does have a story line, "If five members of a cross-crountry team finished in consecutive order, and their team score was 45, find the place of each runner." Or, for example, "A rectangle has an area of 80 square meters and its length is 2 meters more than its width. What is the perimeter of the rectangle?" has no story line; the following problem does have a story line, "Mr. Smith wants to fence his rectangular vegetable garden. His garden contains 80 square meters and its length is 2 meters more than its width. How many meters of chain fencing must he buy?" Finally, "Divide 30 into two parts such that one part is 4 more than 4 times the other part," is not a story problem; the following problem is a story problem: "The entertainment portion of a 30-minute TV program lasted 4 times longer than 4 times the portion devoted to advertising. How many minutes were devoted to advertising and to entertainment?" (5) The story line was more complicated than an arithmetic word problem, i.e., the problem required more than a chain of addition and/or subtraction. For example, "Tom has 4 pencils. Then he gives 2 pencils away, and he finds 3 more pencils. How many pencils does he have now?" was not systematically included in the data base. Generally, arithmetic word problems are covered earlier in the curriculum, and thus are not emphasized in secondary school textbooks. However, some arithmetic word problems were included in order to provide a general overview of the types of story problems used in textbooks.

Problem Format: Selection of Algebra Story Problems

The foregoing procedure generated approximately 1500 cards, with an algebra problem on each. Although the main goal was to select only algebra story problems, many non-story problems were included in the data base. The reasons were: (1) In order to guard against missing any story problems, all problems that seemed even remotely to be story problems were included with a more detailed inspection made later. (2) A substantial number of representative non-story problems were included in order to provide a general overview of all the problems in algebra textbooks. Thus, the data base consisted of all algebra story problems and some representative non-story problems.

The next step was to sort the problems into five mutually exclusive groups based on problem format. Table 1 presents definitions and examples of the groups. First, problems can be divided into story and non-story groups based on the criteria given above; then, within each of these groups, the problems can be divided into those based on a source formula and those that are not. For story problems, most problems are based on a formula involving rate (such as rate x time = distance), geometry (such as area = length x width), physics (such as force = weight x distance), or statistics (such as the formula for number of combinations); some story problems such as number-story problems do not rely on a source formula. For non-story problems, most problems do not require a source formula (e.g., arithmetic word, number, equation); but some non-story problems are based on a source equation (formula).

Insert Table 1 about here

Classification of Story Problems by Family and Source Formula

The foregoing procedure, based on the definitions in Table 1, allowed for the selection of 1097 algebra story problems. Of the story problems, some were based on a simple formula (such as distance = rate x time) while others were not. There were eight major families of formulas involved in story problems and one family with no source formula:

time rate--These are problems based on a rate formula involving time such as "distance = rate x time" or "output = rate x time."

unit cost rate--These are problems based on rate formula involving unit costs such as "total cost = unit cost x number of units."

percent cost rate--These are problems based on a rate formula involving a percentage of total cost, such as "interest = interest rate x principal" or "profit = cost x markup rate."

straight rate--These are problems based on a rate formula in which one amount (or number) is compared to another as a simple rate, percent, fraction, index, proportion, or ratio.

geometry--These are problems based on simple formulas from geometry such as area and perimeter of rectangles, "area = length x width"; circumference of circles, $C = \pi r^2$; and the Pathahorean law for right triangles, $a^2 + b^2 = c^2$.

physics--These are problems based on simple physics laws such as Ohm's Law, $R = V|I$.

statistics--These are problems based on simple statistical formulas such as the formula for number of combinations, $C = N!/(N-r)!r!$.

number-story--These are story problems that are not based on any source formula.

Examples of typical source formulas are given in Table 2. The major families and source formulas for story problems in the sample are shown in the top of Table 3.

Insert Tables 2 and 3 about here

Classification of Story Problems by Category

In the foregoing analysis, each problem was classified according to its source formula. This helped create several major "families" of problems, all sharing the same kind of source formula. Within each family there were several categories of problems. The bottom of Table 3 lists the major categories within each family. As can be seen, some of the categories are "simple"--i.e., they directly involve only the source formula--while others are "complex"--i.e., they use the source formula in a more complex larger equation. Definitions and examples of some of the major simple categories are given in Table 2. Of the 1097 story problems, 199 fit "simple categories," leaving 898 story problems in "complex categories." The complex categories listed in the bottom of Table 3 as well as the simple categories listed in the middle of Table 3, summarize all of the problem categories explicitly described in the textbooks as well as all of the problem categories listed by Rich (1973)--i.e., consecutive integer, age, ratio, angle, perimeter, coin, mixture, investment/interest, motion, work, combination, digit, statistics--and all of the problem categories listed by Hinsley, Hayes & Simon (1977) except scale conversion.[2]

Many of the simple and complex categories in Table 3 contain only story problems--i.e., most of the rate-based problems are "pure" in the sense that they always require a story line. However, each of the category names followed by an asterisk in Table 3 may contain some story and some non-story problems. Table 4 shows examples of story and non-story versions within each of these categories, although the present analysis counted only the story problems. It should be noted that for most of the problems in Table 4, there were far more non-story problems. For example, for area problems non-story problems outnumbered story problems at the ratio of approximately 10 to 1; similarly, for consecutive integer problems the non-story problems outnumbered the story problems at the ratio of approximately 12 to 1. Some problems were never presented in story form; these are digit, angle, and number problems, as exemplified in Table 4. Although there were very large numbers of each of these categories and although there are many varieties of each category, the full number of these categories were not selected for the present sample.

Insert Table 4 about here

Classification of Story Problems by Template

The foregoing section produced a list of simple and complex category names that represent the most common level of analysis. However, it must be noted that not all problems in a given category are similar. For example, it is possible to locate 12 types of motion problems, with some types involving three variations. Therefore, this section explores a more detailed level of classification of problems within each category--classification by template.

A template refers to a specific propositional structure and story line. For purposes of the present analysis we break each story problem into a list of propositions. The units that make up propositions are:

variables--such as "the time to go upstream" operators--such as "is twice the time"

values--such as any number

relations--such as "equal to"

An analysis of the propositional structure of story problems revealed that there are three major kinds of propositions:

assignment of a value to a variable, such as "A boat travels upstream in 2 hours" could be represented as "time to go upstream = 2," or in general form as "time A = ___"

assignment of a relation between two variables, such as "The length is 2 meters greater than the width" could be represented as "length = 2+ width," or in general form as "length = <u>REL</u> width"

assignment if a variable to an unknown value, such as "Find the speed in still water" could be represented as "speed in still water = X", or in general form as "speed in still water = FIND."

The basic information in any problem can be represented as a list of proportions, consisting of any number of each of the above three types of propositions, and with each proposition consisting of some combination of variable, operator, value, and/or relation.

For example, consider the problem, "A boat travels 8 miles upstream against the current in the same time that it travels 12 miles downstream with the current. If the rate of the current is 2 mph, what is the speed of the boat in still water?" This problem can be expressed as:

distance upstream = ___

distance downstream = ___

rate of current = ___

rate in still water = FIND

Or consider the problem, "Working together, Mary and Jane can do a job in 5 months. It takes Mary twice as long as Jane to do the job alone. How long would it take Mary working alone?" The problem can be expressed as:

rate for Mary and Jane together = ______

rate for Mary = REL rate for Jane

rate for Mary = FIND

I refer to the list of propositions and a statement of the story line as the template for a problem. Problems belong to the same template if they share the same story line and same list of propositions, regardless of the actual values assigned to each variable, the actual relation assigned to a pair of variables, or which variable is assigned to the unknown. Thus, for example, the following problem belongs to the same template as the work problem above: "Working alone, Mary can do a job in 5 months. Mary takes half as long as Jane to do her job alone. How long will it take if Mary and Jane work together?" In this problem the values are different and the unknown is assigned to the joint rate rather than the individual rate, but the propositions are of the same form. The following problem involves a different template "Mary can do a job in 5 hours and Jane can do the job in 4 hours. If they work together, how long will it take to do the job?" This involves a different template because there is no "relation" proposition, i.e., the propositions are:

rate for Mary = _____

rate for Jane = _____

rate for Mary and Jane together = FIND

As can be seen, templates express the specific propositional structure of a problem.

For any template, the unknown could be assigned to any of the listed variables. Thus, for the work problem given immediately above there are three variations for the template: rate for Mary = 5, rate for Jane = 4, rate for Mary and Jane together = X; rate for Mary = 5, rate for Mary and Jane together = 2 2/9, rate for Jane = X; rate for Jane = 4, rate for Mary and Jane together = 2 2/9, rate for Mary = x. Each of these represent a variation of the "work together" template.

In addition, modifications may be introduced for any template. For example, instead of giving rates the problem might say: "Mary works from noon until 5 p.m. on the job. If Jane helps her they can finish by 2:00 p.m. When would Jane finish if she started at noon and worked alone?" This problem is a modification of the above template because it involves the same propositional structure, but with the need to convert clock readings to absolute times.

Table 5 provides a list of each template found for each of the simple and complex categories. Since simple categories consist of just one template, the main focus of this section is on the templates within each complex category. The numbers in parentheses refer to the frequencies with which each template was observed in the sample. For example, there were 113 motion problems; 20 were the simple category (simple DRT), but there were 11 other templates ranging in frequency from 23 for overtake problems to 3 for same direction problems. Table 6 presents a more detailed description of each template, along with variations and modifications that were observed. For each template, the following information is given in Table 6: the name of the category, the name of the template, the frequency of the template, a description of the story line (although a variety of characters or objects are involved in some cases), a list of the propositions, a notation of variations (i.e., which variables are unknowns), a notation of modifications, an example problem. As can be seen, there are approximately 90 templates represented in the sample. However, this number is cut in half if we focus only on the templates that occur at least 10 times in the sample.

Insert Tables 5 and 6 about here

Implications for Research and Instruction

Development of problem schemata. The importance of understanding of story problems at the level of templates can been seen when one attempts to solve problems. For a novice, all motion problems may look alike, but the solution procedure is different for different templates. Many frustrations and disappointments may arise when a student attempts to apply a solution procedure to one template of a motion problem when actually the problem belongs to a different template. A failure to explicitly describe each template--or presentation of only one or two templates in a category that contains many--may lead to the development of a problem solving approach that is too narrow. Templates can be used in instruction in a variety of ways, such as systematically moving from one template to another to encourage transfer skills.

Greeno and his colleagues (Heller & Greeno, 1978; Riley & Greeno, 1978) have provided in-depth analyses of some arithmetic work problems, offering "schemas" which are similar to our templates for story problems. Recent research by these investigators suggest that some templates are much more difficult than others, even when the same computations are involved. There also appears to be a developmental trend in children's ability to deal with various types of problems. Further research and development is needed in order to take full advantage of the "template level of analysis."

Several groups of researchers have shown that students try to find out what "type" of problem is presented and then use a solution strategy appropriate for that type (Hinsley, Hayes, & Simon, 197 ; Heller & Greeno, 1978; Riley & Greeno, 1978). However, errors occur when students assimilate a problem to an inappropriate schema, such as thinking a motion problem is a current problem. Additional research is required to determine how subjects make judgments concerning problem types, i.e., what are the features of the problem that are most salient for beginners and for more advanced students. A related issue concerns the effects of explicit instruction concerning problem types and templates.

<u>Pattern matching practice</u>. Simon (1980) has argued that algebra instruction emphasizes the algebraic operations (such as adding a constant to both sides of an equation, etc.), but often ignores teaching when to apply the operators. Good problem solvers tend to learn the conditions for each operation by practicing and by examining worked out problems. However, an important research question concerns whether some students should be given more practice in recognition of patterns. For example, does explicit instruction and practice in recognizing different templates for the same problem category lead to more efficient learning?

<u>Transfer to different problem types</u>. In the present taxonomy there are many cases of problem isomorphs, i.e., problems in which the solution paths map directly onto one another in one-to-one fashion (see Hayes & Simon, 1976). For example, the <u>Motion: Opposite Direction</u> templates is isomorphic to the <u>Work: Together</u> template. Research using traditional problems such as Tower of Hanoi (Hayes & Simon, 1976) or Missionaries & Cannibals (Reed, Ernst & Banerji, 1974) indicates that transfer from one form of the problem to an isomorphic

form is often quite difficult. Additional research is required to determine how subjects transfer from one version of an algebra story problem to another, and in particular, to determine what variables influence ease of transfer. For example, one question is whether practice on one type of problem at a time (as is presently encouraged in most textbooks) inhibits transfer as compared to practice of a mixture of problem types. A related question concerns transfer to creative problem solving. For example, what experiences enhance performance when unusual story problems are presented?

Footnotes

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[1] I wish to thank the staff of the Curriculum Library, University of California, Santa Barbara, for assistance on this project.

[2] The "scale conversion" problem asks for a formula as the answer and thus does not fit our criteria as a story problem. However, in a slightly modified form it would fall into the "direct variation" category.

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Table 1

Five Formats of Problems

Type	Definition	Example
Equation	One or more equations are presented, containing one or more unknowns. The task is to solve for an unknown.	$X + 10 = 2X - 2$. Solve for X.
Formula	One or more equations are given, along with values for some of the variables. The task is to solve for an unknown.	Use the formula $R = V/I$. $R = 10$, $I = 2$, Find V.
Number	One or more sentences are presented, containing one or more unknowns, with no story line. The task is to solve for an unknown.	Two less than twice is the same as that number added to 10. Find the number.
Arithmetic Word	A story line involving simple addition and/or subtraction is given.	Tom has 5¢. He spends 2¢. He then earns 10¢. How much does he have now?
Story	A story line with characters, actions, and/or objects.	A boat travels 8 miles upstream against the current in the same time that it travels 12 miles downstream with the current. If the speed of the current is 3 mph, what is the speed of the boat in still water?

Table 2

Some Common Source Formulas for Story Problems

Name	Formula	Example of Simple Problem
Simple DRT	distance = rate x time	If a car travels 10 hours at 30 miles per hour, how far will it go?
Simple Work	output = rate x time	If a machine can produce 10 units per hour, how many units can be produced in an 8 hour day?
Simple Unit-Cost	total cost = unit cost x number of units	If pencils cost 5¢ each, how much will a dozen pencils cost?
Simple Interest	interest = interest rate x principal	How much will be earned if $1000 is invested at 8% interest for 1 year.
Simple Profit	profit = markup rate x cost	If a TV set costs the seller $300 and the markup if 20%, how much profit will be made?
Simple Discount	discount = discount rate x cost	A TV set regularly sells for $400. A certain store is offering 25% off the regular price. How much can you save?
Simple Percent	amount-1 = rate x amount-2.	Of 300 votes cast in an election, Tom received 30% of the votes. How many votes did he get?

Table 3

Families, Simple Categories and complex Categories for Story Problems

FAMILY	AMOUNT-PER-TIME RATE (TIME RATE)	COST-PER-UNIT RATE (UNIT COST RATE)	PORTION-TO-TOTAL COST RATE (PERCENT COST RATE)	AMOUNT-TO-AMOUNT RATE (STRAIGHT RATE)	NUMBER-STORY	GEOMETRY	PHYSICS	STATIST
SIMPLE CATEGORY (Source Formula)	SIMPLE DRT SIMPLE WORK	SIMPLE UNIT-COST	SIMPLE INTEREST SIMPLE PROFIT SIMPLE DISCOUNT	SIMPLE RATE * SIMPLE PERCENT* SIMPLE FRACTION* SIMPLE PROPORTION* SIMPLE INDEX* SIMPLE RATIO*		SIMPLE AREA* SIMPLE PERIMETER* SIMPLE CIRCUMER* SIMPLE PATHAGOREA* SIMPLE TRAPEZOID*	FALLING BODY* OHM'S LAW* OTHER*	PERMUTA PROBABI
COMPLEX CATEGORIES	MOTION CURRENT WORK	FIXED COST COIN DRY MIXTURE	INTEREST/INVESTMENT PROFIT DISCOUNT	DIRECT VARIATION INVERSION VARIATION WET MIXTURE	PART* AGE* CONSECUTIVE INTEGER* DIGIT** ANGLE** NUMBER**	AREA* FRAME PERIMENTER	FALCRUM*	PROGRESSION EXPONENTIAL MAXIMIZATIO

NOTE.--Single asterisk (*) indicates some problems are not story problems.

Double asterisk (**) indicates all problems are not story problems.

Table 4

Some Story and Non-Story Problems

Name	Story Version	Non-Story Version
Simple Percent	John received 40% of the 200 votes cast in an election. How many votes did he receive?	What is 40% of 200
Part	A 3 foot board is cut into pieces such that one piece is twice as long as the other. How long is each piece?	A certain number plus twice that number is equal to 3.
Age	Pete is twice as old as his sister. In 2 years he will be 2 years older than his sister. How old is he now?	Half of John's age is the same as his age minus 2. What is his age?
Consecutive Integer	In a race three runners finished in consecutive order and earned a total of 15 points. What are their places?	Find three consecutive integers such that their sum is 15.
Digit	(none)	Find a two digit number such that the first digit is twice the second and their sum is 6.

Table 4 (Continued)

Name	Story Version	Non-Story Version
Angle	(none)	A right angle is divided into two smaller angles. One angle is 15 more degrees than the other. How large is it?
Number	(none)	Find a number such that 3 more than double the number is equal to 23.
Rectangle	John wants to cover his window with travel stickers. Each sticker is 1 inch by 1 inch. The length of the window is twice as great as the width, and the perimeter is 36 inches. How many stickers does John need?	Find the area of a rectangle if the length is twice as great as the width and the perimeter is 36 inches.

Table 5a

Categories and Templates for the Amount-Per-Time Family

AMOUNT-PER-TIME RATE (293)

COMPLEX CATEGORY	MOTION (113)	CURRENT (49)	WORK (106)
SIMPLE CATEGORY (TEMPLATE)	SIMPLE DRT (20)	(same)	SIMPLE WORK PLATE (5)
TEMPLATES	OVERTAKE (23)	TOTAL TIME 1 (13)	TOGETHER ABSOLUTE (49)
	OPPOSITE DIRECTION (23)	ROUND TRIP ABSOLUTE 1 (10)	INDIVIDUAL ABSOLUTE (25)
	ROUND TRIP 1 (13)	ROUND TRIP ABSOLUTE 2 (10)	INDIVIDUAL RELATIVE (12)
	CLOSURE 1 (12)	EQUAL TIME (9)	FINISH JOB TOGETHER (6)
	SPEED CHANGE 1 (10)	PART (2)	THREE TOGETHER (6)
	EQUAL TIME (8)	ROUND TRIP RELATIVE 1 (2)	THREE INDIVIDUAL (5)
	EQUAL DISTANCE (7)	ROUND TRIP RELATIVE 2 (1)	FINISH JOB INDIVIDUAL (2)
	TRIANGLE (4)	ROUND TRIP RELATIVE 3 (1)	TOGETHER RELATIVE (1)
	ROUND TRIP 2 (4)	TOTAL TIME	
	CLOSURE 2 (4)		
	SAME DIRECTION (3)		
	SPEED CHANGE 2 (2)		

Note.--Numbers in parentheses in Tables 5a-5g indicate observed frequencies in data base.

Table 5b

Categories and Templates for Cost-Per-Unit Family

	COST-PER-UNIT RATE (175)		
COMPLEX CATEGORY	UNIT COST (32)	COINS (70)	DRY MIXTURE (60)
SIMPLE CATEGORY (TEMPLATE)	SIMPLE UNIT COSTS (13)	(same)	(same)
TEMPLATES	FIXED PLUS UNITS (32)	TOTAL NUMBER GIVEN (32)	TWO ABSOLUTE AMOUNTS (25)
		RELATIVE NUMBER GIVEN (31)	TWO RELATIVE AMOUNTS (25)
		THREE RELATIVE NUMBERS (7)	ADD TO GIVEN (8)
			THREE RELATIVE AMOUNTS (2)

Table 5c

Categories and Templates for Portion-of-Total Family

PORTION-OF-TOTAL COSTS (147)			
COMPLEX CATEGORY	INTEREST/INVESTMENT (99)	PROFIT (-)	DISCOUNT (-)
SIMPLE CATEGORY (TEMPLATES)	SIMPLE INTEREST (9)	SIMPLE PROFIT (4)	SIMPLE DISCOUNT (7)
	SIMPLE INTEREST & TIME (14)	SIMPLE COST (6)	SIMPLE COST (8)
TEMPLATES	COMPOUND TIME (56)		
	TWO ABSOLUTE AMOUNTS (19)		
	TWO RELATIVE AMOUNTS (11)		
	TWO EQUAL INTEREST AMOUNTS (8)		
	THREE RELATIVE AMOUNTS (3)		
	DEPRECIATION (2)		

Table 5d

Categories and Templates for Amount-Per-Amount Family

AMOUNT-PER-AMOUNT (276)			
COMPLEX CATEGORY	DIRECT VARIATION (83)	INVERSE VARIATION (33)	WET MIXTURE (60)
SIMPLE CATEGORIES (TEMPLATES)	SIMPLE RATE (8) SIMPLE PERCENT (26) SIMPLE RATIO (17) SIMPLE FRACTION (16) SIMPLE INDEX (12) SIMPLE PROPORTION (11)	(same)	(same)
TEMPLATES	MISCELLANEOUS (#6) UNIT COST (19) TRAVEL (10) MAP SCALE (10) WEIGHT (8)	PRESSURE-VOLUME (10) PHYSICS (7) MOTION (7) WORK (6) UNIT-COST (3)	ADD TO GIVEN (42) TWO ABSOLUTE AMOUNTS (18)

Note.--All bracketed names for DIRECT VARIATION refer to same single template; all bracketed names for INVERSE VARIATION refer to same angle template. Names in brackets refer to different situations, but share the same template.

Table 5e

Categories and Templates for Number-Story Family

	NUMBER-STORY (90)		
COMPLEX CATEGORY	PART (48)	AGE (38)	CONSECUTIVE INTEGER (4)
SIMPLE CATEGORY (TEMPLATE)			
TEMPLATES	NUMBER (*) TWO PIECE RELATIVE (41) THREE PIECE RELATIVE (7)	NUMBER (*) RELATIVE THEN NOW (28) ABSOLUTE THEN NOW (10)	NUMBER (*) SUM (4)

Note.--Other non-story categories are: NUMBER, DIGIT, RATIO, ANGLE.

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Table 5f

Categories and Templates for Geometry Family

	GEOMETRY (69)		
COMPLEX CATEGORY	RECTANGLE (46)	CIRCLE (7)	TRIANGLE (16)
SIMPLE CATEGORY (TEMPLATE)	SIMPLE AREA (10)	SIMPLE CIRCUMFERENCE (7)	SIMPLE PYTHAGOREAN (16)
TEMPLATES	FRAME ABSOLUTE 1 (11)		
	RELATIVE AREA (10)		
	RELATIVE PERIMETER (8)		
	FRAME ABSOLUTE 2 (5)		
	FRAME RELATIVE 1 (1)		
	FRAME RELATIVE 2 (1)		

Table 5g

Categories and Templates for Physics Family and Statistics Family

	PHYSICS (17)	STATISTICS (30)	
COMPLEX CATEGORY	FULCRUM (17)	EXPONENTIAL (22)	PROGRESSION (8)
TEMPLATES	TWO ABSOLUTE WEIGHTS (14)	DECAY (10)	INCREMENT-DECREEMENT (8)
	TWO RELATIVE WEIGHTS (2)	REBOUND 1 (7)	
	THREE WEIGHTS (1)	REBOUND 2 (5)	

Note.--Other non-story PHYSICS problems are: FALLING BODIES, OHN'S LAW, OTHER PHYSICS.

Other STATISTICS problems are: PERMUTATIONS/COMBINATIONS, PROBABILITY, MAXIMIZATION.

Table 6

Templates for Each Category of Problem

Motion: Simple DRT (n = 20)

A vehicle travels a certain distance in certain amount of time at a certain speed.

distance = _____

time = _____

rate = FIND

Variations: rate unknown (12), time unknown (5), distance unknown (3).

Bill Less drove from Boston to Cleveland, a distance of 624 miles, in the time of 12 hours. Find his rate of driving.

Motion: Overtake (n = 23)

One vehicle starts and is followed later by a second vehicle that travels over the same route at a faster rate.

rate for A = _____

rate for B = _____

time for A and B = _____

time for B to overtake A = FIND

Variations: time to overtake unknown (13), rate for A or B unknown (5), distance traveled unknown (5).

A train leaves a station and travels east at 72 km/h. Three hours later a second train leaves on a parallel track and travels east at 120 km/h. How long will it take to overtake the first train?

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Table 6 (continued)

Motion: Opposite Direction (n = 23)

Two vehicles leave the same point traveling in opposite directions.

rate for A = ____

rate for B = ____

distance between A and B = ____

time = FIND

Variations: time unknown (16), rate unknown (7).

Modification: vehicle B starts after vehicle A (3).

Two trains leave the same station at the same time. They travel in opposite directions. One train travels 64 km/h and the other 104 km/h. In how many hours will they be 1008 km apart?

Motion: Round Trip (n = 13)

A traveler (or vehicle) travels from point A to point B and returns.

rate from A to B = ____

rate from B to A = ____

time for entire trip = ____

distance for entire trip = FIND

Variations: distance unknown (10), time unknown (3).

Modification: Delay before starting on return (3).

George rode out of town on the bus at an average speed of 20 miles per hour and walked back at an average speed of 3 miles per hour. How far did he go if the entire trip took six hours?

Table 6 (continued)

Motion: Closure 1 (n = 12)

Two vehicles start at different points traveling towards one another.

rate for A = _____

rate for B = _____

distance between A and B = _____

time = FIND

Variations: time unknown (10), rate unknown (2), find distance (0).

Modifications: Vehicle B starts after A (4).

Two bikers start at the same time from towns 36 miles apart. The bikers move toward each other; one travels at 4 mph and the other at 8 mph. How long will it take for them to meet?

Motion: Speed-Change 1 (n = 10)

A vehicle travels at a certain rate for the first leg of a trip and then changes to another rate for the remainder of the trip.

rate for first part of trip = _____

rate for second part of trip = _____

total distance = _____

total time = _____

distance for first part (and/or second part) = FIND

variations: time unknown (3), distance unknown (7)

G. Otrotrot jogs and walks to school each day. He averages 4 km/h walking and 8 km/h jogging. The distance from home to school is 6 km and he makes the trip in 1 hour. How far does he jog in a trip?

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Table 6 (continued)

Motion: Equal-Times (n = 8)

One vehicle travels a certain distance at a certain rate in the same time that a second vehicle travels a certain distance a certain rate.

distance for A = _____

distance for B = _____

rate for A = REL rate for B

rate for A (and/or rate for B) = FIND

Modifications: While one goes X distance, other goes Y distance (2).

A car travels 300 kilometers in the same time that a train travels 200 kilometers. The speed of the car is 20 kilometers per hour more than the speed of the train. Find the speed of the car and the speed of the train.

Motion: Equal-Distance(n = 7)

One vehicle travels a certain amount of time at a certain rate and covers the same distance as a second vehicle that travels for a certain amount of time at a certain rate.

rate for A = _____

rate for B = _____

time for A = REL time for B

distance traveled by A (or B) = FIND

Variations: distance unknown (3), rate unknown (2), time unknown (1).

Modification: While one travels in X time, other travels in Y time (1).

An express train travels at 80 km/h from Baysville to Seneca. It takes 2 hours less for the trip than for a passenger train that travels 48 km/h. How far apart are Seneca and Baysville?

Table 6 (continued)

Motion: Triangle (n = 4)

Two vehicles leave same point at same time and travel at right angles to one another.

rate for A = REL rate for B

time for A and B = ______

distance between A and B = ______

rate for A (and/or rate for B) = FIND

Two joggers leave the same point at right angles to one another. One travels 1 km/h faster than the other. After 2 hours they are 10 km apart. Find the speed of each.

Motion: Round-Trip 2 (n = 4)

A traveler or vehicle travels from point A to point B and returns.

rate from A to B = ______

rate from B to A = ______

time from A to B = REL time from B to A

distance from A to B = FIND

Variations: distance unknown (2), rate unknown (2).

Polly Paddle has just enough money to rent a canoe for 2! hours. How far out on the lake can she paddle and return on time if she paddles out at 3 km/h and back at 2 km/h?

Motion: Closure 2 (n = 4)

Two vehicles start at different points traveling towards one another.

distance = ______

time = ______

rate for A = REL rate for B

Table 6 (continued)

rate for A (and/or B) = FIND

Two hikers start at the same time from towns 36 miles apart. The hikers move towards each other and meet in 3 hours. One hiker is going twice as fast as the other. What is the rate of each hiker?

Motion: Same-Direction (n = 3)

Two vehicles leave same point at same time traveling in same direction at different rates.

rate for A = _____

rate for B = _____

distance between A and B = _____

time for A and B = FIND

Variations: time unknown (1), distance unknown (1), rates unknown (1).

Motion: Speed-Change 2 (n = 2)

A vehicle travels at a certain rate for the first leg of a trip and then changes to another rate for the remainder of the trip.

rate for first part = REL rate for second part

total distance = _____

time for first part = _____

time for second part = _____

rate for first part (and/or rate for second part) = FIND

Current: Total -Time-1

A boat travels a certain distance with the current and a certain distance against the current in a total time.

rate of current = _____

Table 6 (continued)

distance with current = _____

distance against current = _____

total time = _____

rate in still water = FIND

Variations: rate in still water unknown (6), rate of current unknown (7).

The current in a stream moves at a speed of 4 km/h. A boat travels 4 km upstream and 12 km downstream in a total time of 2 hours. What is the speed of the boat in still water?

Current: Round-Trip 1 (n = 13)

A boat travels a certain distance with the current and a certain distance against the current in a total time.

rate of current = _____

distance with current = _____

distance against current = _____

total time = _____

rate in still water = FIND

Variations: rate in still water unknown (6), rate of current unknown (7).

The current in a stream moves at a speed of 4 km/h. A boat travels 4 km upstream and 12 km downstream in a total time of 2 hours. What is the speed of the boat in still water?

Current: Round-Trip 1 (n = 10)

A boat travels with the current in a certain time and returns against the current in a certain amount of time.

time with current = _____

time against current = _____

Table 6 (continued)

rate of current = _____

rate in still water = FIND

Modifications: story line with boats and airplanes.

A boat travels 3.15 hours downstream, where the current is 5.82 km/h. It returns in 9.97 hours. Find the speed of the boat in still water.

Current: Round-Trip-2 (n = 10)

A boat travels with the current in a certain time and returns in a certain time against the current.

time with current = _____

distance one-way = _____

rate in still water (and/or rate of current) = FIND

Modifications: story line with boats and airplanes.

Variations: rate in still water unknown (9), distance one-way unknown (1).

Fly High Airlines flies from Podunk to Swampville in 5 hours with a tailwind. The return trip, against the same wind, takes 6 hours. Podunk is about 5550 km from Swampville. Find the speed of the plane and the velocity of the wind.

Current: Equal-Time (n = 9)

A boat travels a certain distance with a current in the same time it can travel a certain distance against the current.

rate in still water = _____

distance with current = _____

distance against current = _____

rate of current = FIND

Variations = rate in still water unknown (3), rate of current unknown (6).

Table 6 (continued)

A boat travels at a rate of 15 kilometers per hour in still water. It travels 60 kilometers upstream in the same time that it travels 90 kilometers downstream. What is the rate of current?

Current: Part (n = 2)

A boat travels at a certain rate with the current and a certain rate against the current.

rate with current = _____

rate against current = _____

rate in still water (and/or rate of current) = FIND

Fairfield's rowing team can row downstream at a rate of 7 mph. They can row back to the starting point at a rate of 3 mph. Find their rowing rate in still water and the rate of the current.

Current: Round-Trip-Relative 1 (n = 2)

The time to travel with the current is compared to the time to travel the same distance against the current.

rate in still water = _____

rate of current = _____

time with current = REL time against current

distance one-way = FIND

Variations: rate of current unknown (1), distance one-way unknown (1).

The air speed of an airplane is 225 miles per hour. Flying from city A to city B, it has a tailwind of 25 miles per hour. It takes 3 hours longer to fly from B to A than from A to B. How far apart are the two cities?

Table 6 (continued)

Current: Round-Trip-Relative-2 (n = 1)

The time to travel with the current is compared to the time to travel the same distance against the current.

rate with current = ____

rate against current = ____

time with current = REL time against current

distance one-way = FIND

A ship can go downstream from town A to town B at 32 kilometers per hour in five hours less time than it takes to go upstream from B to A at 24 kilometers per hour. How far apart are the towns?

Current: Round-Trip-Relative-3 (n = 1)

The time and rate to travel with the current is compared to the time and rate to travel against the current.

distance one-way = ____

rate with current = REL rate against current

time with current = REL rate against current

rate with current (and/or rate against current) = FIND

An airplane flew a round-trip training flight from airport A to airport B. The distance between the two airports was 1200 miles. Going against the wind the pilot flew 60 miles per hour slower than returning. It took one hour more time going than returning. What was the speed going and returning?

Current: Total-Time-2 (n = 1)

A boat travels a certain distance with the current and a certain distance against the current in a total time.

rate in still water = ____

Table 6 (continued)

rate of current = _____

time for two-way trip = _____

distance for one-way = FIND

Tim can average 12 mph with his boat in still water. In a river with a current of 4 mph, it takes him 9 hours to travel from point A to point B and return. Find the distance from A to B.

Work: Simple Work (n = 5)

A worker produces a certain output by working at a certain rate for a certain amount of time.

rate of work = _____

time = _____

output = FIND

Variations: output unknown (3), time unknown (2).

A fisherman can catch and clean a fish every 20 minutes. If he spends an 8 hour day fishing, how many fish will he bring home?

Work: Together (n = 49)

Two working together; given individual rates, find combined rate.

rate for A = _____

rate for B = _____

rate for A and B together = FIND

Mary can do a job in 5 hours and Jane can do the job in 4 hours. If they work together, how long will it take to do the job?

Work: Individual (n = 25)

Two working together; given combined rate and rate for one, find rate for other.

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Table 6 (continued)

rate for A = _____

rate for A and B together = _____

rate for B = FIND

To do a job alone, it would take Jane 1! hours. If Mary helps they can do the job in 1 hour. How long would it take Mary to do the job working alone?

Work: Individual-Relative (n = 12)

Two working together; given combined rate and relative individual rates, find individual rates.

rate for A = REL rate for B

rate for A and B together = _____

rate for A (or B) = FIND

Working together, Mary and Jane can do a job in 5 months. It takes Mary twice as long as Jane to do the job alone. How long would it take Mary working alone?

Work: Finish-Job-Together (n = 6)

One worker begins and is joined later by another; given individual rates and time of one on job alone, find combined rate for rest of job.

rate for A = _____

rate for B = _____

time A works alone = _____

rate to complete job for A and B working together = FIND

Mary can do a job in 7 hours and Jane can do it in 5 hours. How long will it take both to finish the job after Mary has been working alone for 3! hours?

Work: Three-Together (n = 6)

Three working together; given individual rates, find combined rate.

rate for A = _____

Table 6 (continued)

rate for B = _____

rate for C = _____

rate for A and B and C together = FIND

To do a job alone, it would take Jane 3 hours, Mary 5 hours and Jerry 6 hours. How long would it take if they all worked together?

Work: Three-Individual (n = 5)

Three working together; given combined rate, and two individual rates, find other rate.

rate for A = _____

rate for B = _____

rate for A and B and C together = _____

rate for C = FIND

Mary can do a job in 3 weeks and Jane can do it in 5 weeks. How long would it take Jerry, if working together all three can do the job in 1 week?

Work: Finish-Job-Individual (n = 2)

One worker begins and is joined later by another; given combined rate and rate for one, find rate for other.

rate for A = _____

time A works alone = _____

rate to complete job when working with B = _____

rate for B = FIND

Jane can do a job in 5 hours. After working for 2 hours she is joined by Mary. Together they finish the job in 1 hour. How long would it take Mary to do the entire job working alone?

Table 6 (continued)

Work: Together-Relative (n = 1)

Two working together; given rate for one, and relative rate for other, find combined rate.

rate for A = ____

rate for B = REL rate for A and B together

rate for A and B together = FIND

Mary can do a job in 3 minutes. It takes Jane 4 minutes longer to do the job than it takes both of them working together. How long does it take to do the job if both work together?

Unit-Cost: Cost-Unit-Total (n = 13)

A certain unit cost for a certain number of units yields a total cost.

number of units = ____

total cost = ____

unit cost = FIND

Variations: unit-cost unknown (10); total unknown (2); number of units unknown (1).

Jean worked 5 hours. She earned a total of $15. How much does she earn each hour?

Unit-Cost: Fixed-Plus-Units (n = 32)

Total cost (or wage) is based on a flat cost plus a certain unit cost applied to a certain number of units.

flat cost = ____

unit cost = ____

number of units = ____

total cost = FIND

Variations: total cost unknown (23); unit cost unknown (7); flat cost unknown (2).

Table 6 (continued)

Sixteen balls of yarn can be bought from a mail order house for 29¢ each plus $2.72 for postage. What does the total order cost?

Coins: Total-Number-Given (n = 32)

A certain number of coins, consisting of two different types of coins, totals a certain total amount.

number of coins = ____

value of coin A = ____

value of coin B = ____

total value of all coins = ____

number of coin A (and/or number of coin B) = FIND

Modifications: story lines about coins, stamps, tickets, items sold in store.

A collection of 25 dimes and quarters amounts to $5.05. How many of each kind of coin are there?

Coins: Relative-Number-Given (n = 31)

In a collection of two types of coins, the number of one type is related to the number of the other type and the collection is worth a certain total amount.

number of coin A = REL number of coin B

value of coin A = ____

value of coin B = ____

total value of all coins = ____

number of coin A (and/or number of coin B) = FIND

Modifications: story lines about coins, stamps, and tickets

Ken's coin collection contains 7 more dimes than nickels. In all, the collection amounts to $3.25. How many of each coin does he have?

Table 6 (continued)

Coin: Three-Relative-Numbers (n = 7)

In a collection of three types of coins, the number of one type is related to the number of second type and the number of the second type is related to the number of the third type, and the collection is worth a certain total amount.

number of coin A = REL number of coin B

number of coin B = REL number of coin C

value of coin A = _____

value of coin B = _____

value of coin C = _____

total value of all coins = _____

number of coin A (and/or number of coin B, and/or coin C) = FIND

Jill has some pennies, nickels, and dimes. In all she has $3.92. The number of nickels is two less than the number of pennies. She has 13 more dimes than pennies. How many pennies, how many nickels, and how many dimes does she have?

Dry Mixture: Two-Absolute-Amounts (n = 25)

Some amount of one item with a certain unit cost is mixed with some amount of another item with a certain unit cost to yield a total amount with a certain unit cost; individual amount unknown.

unit cost for A = _____

unit cost for B = _____

total amount for mixture = _____

unit cost for mixture = _____

number of units of A (and/or number of units of B) = FIND

Modification: story line with price per pound, per metric unit, or per piece.

Table 6 (continued)

A grocer mixes peanuts worth $1.65 a pound and almonds worth $2.10 a pound. She wants 30 pounds of the mixture worth $1.83 a pound. How many pounds of each should the grocer include in the mixture?

Dry Mixture: Two-Relative-Amount (n = 25)

Some amount of one itme with a certain unit cost is mixed with some amount of another item with a certain unit cost to yield a total amount with a certain total cost; relative amounts are known.

unit cost for A = ____

unit cost for B = ____

total cost for mixture = ____

number of units of A = REL number of units of B

number of units of A* (and/or number of units of B) = FIND

Modifications: story line with price per pound, per metric unit, or per piece.

Pedro wants to mix candy selling at $2.20 per kg with another selling at $2.40 per kg. He wants to make an $11.60 gift box. The number of kg at $2.20 per kg is 1 less than the number of kg at $2.40 per kg. Find the number of kg of each.

Dry Mixture; Add-to-Given (n = 8)

Given a certain amount of one item with a certain unit cost, some amount of another item is added to yield a total amount with a certain unit cost.

unit cost for A = ____

unit cost for B = ____

number of units of A = ____

unit cost for mixture = ____

unit cost B = FIND

Table 6 (continued)

Some corn costing 60¢ per kg is added to 50 kg of oats costing 90¢ per kg to make animal feed costing 75¢ per kg. How many kg of corn should be added?

Dry Mixture: Three-Relative-Amounts (n = 2)

Certain amounts of three itmes with different unit costs are mixed to yield a total amount with a certain total cost.

unit cost for A = _____

unit cost for B = _____

unit cost for C = _____

number of units of A = REL number of units for B

number of units for B = REL number of units for C

total cost of mixture = _____

number of units of A (and/or B, and/or C) = FIND

Chemicals A, B and C cost 60¢, 40¢, and 80¢ per gram, respectively. They are mixed so that the number of grams of B is twice the number of grams of A and is 3 less than the number of grams of C. The mixture is worth $11.40. How many grams of each chemical should be used?

Interest: Simple Interest (n = 9)

A certain interest rate applied to a certain principal yields a certain amount of interest.

amount of interest plus amount of principal = _____	amount of interest = _____
rate of interest = _____	amount of principal = _____
amount of principal = FIND	rate of interest = FIND

Variations: principal unknown (4); rate unknown (3); interest unknown (2).

An investment is made at 6%. It grows to $848 at the end of the year. How much was originally invested?

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Table 6 (continued)

Interest: Simple-Interest-Time (n = 14)

A certain interest rate applied simply to a certain principal for a certain amount of time yields a certain amount of interest.

amount of principal = _____

rate of interest = _____

amount of interest = _____

time of loan = FIND

Variations: time unknown (4); principal unknown (4); rate unknown (3); amount of interest unknown (3).

Sheila borrowed $2700 from the bank at 9°% interest. She paid $499.50 in interest. How long did she keep the loan?

Interest: Compound-Time (n = 56)

A certain interest rate applied compounded to a certain principal for a certain amount of time yields a certain amount of interest.

amount of principal = _____

rate of interest = _____

time of loan = _____

amount of interest plus amount of principal = FIND

Variations: rate unknown (29); interest plus principal unknown (18); principal unknown (5); time unknown (4).

Suppose $750 is invested at 5% compounded annually. What amount will be on the account at the end of two years?

Interest: Two-Absolute-Amounts (n = 19)

A certain amount of money is split into two parts with one part invested at one rate and the other part invested at another interest rate.

Table 6 (continued)

total amount invested = _____

rate for part A = _____

rate for part B = _____

amount of interest from total investment = _____

amount in part A (and/or amount in part B- = FIND

Part of $2000 is invested at 7.5% annual interest. The rest is invested at 6%. Last year's interest was $136.50. How much money was invested at each rate?

Interest: Two-Relative-Amounts (n = 11)

A certain amount of money is invested at one rate and another amount is invested at another rate.

amount in part A = REL amount in part B

rate for part A = _____

rate for part B = _____

amount of interest from total investment = _____

amount in part A (and/or amount in part B) = FIND

Delaine Jones invested a certain amount of money at 6% annual interest and $2000 more than that amount at 8% annual interest. Last year she received $580 in interest. How much did she invest at each rate?

Interest: Two-Amounts-Equal-Interest (n = 8)

A certain amount of money invested at one rate yields the same interest as another amount of money invested at a different rate.

amount in part A = _____

rate for part A = _____

rate for part B = _____

amount in part B = FIND

Table 6 (continued)

Alan Tokashira invested $5000 at 7% interest. How much must he invest at 6.5% interest to obtain the same income?

Interest: Three-Amounts (n = 3)

Part of some is invested at one rate, part is invested at another rate, and part is not invested at all.

rate for part A = ____

amount of part A = REL amount of total

rate for part B = ____

amount of part B = REL amount of total

amount of interest from investments = ____

total amount = FIND

Alba invested one-half of her money at 5.75% interest and one-fourth of her money at 5.5%. If her total interest at the end of one year was $136, find her original sum of money.

Interest: Depreciation (n = 2)

A certain amount depreciates at a certain rate pertinent.

original value = ____

rate of depreciation = ____

current value = ____

number time units = FIND

A piece of machinery valued at $50.000 depreciates 10% per year by the fixed rate method. After now many years will the value have depreciated to $25,000.

Profit: Simple Profit (n = 4)

The amount of tax on an amount is determined by applying the rate to the amount.

high price minus low price = ____

Table 6 (continued)

percent profit = _____

low price = FIND

Variations: low price unknown (2); percent unknown (2).

A 6% excise tax on the value of a car amounts to $180. What is the value of the car?

Profit: Simple Cost (n = 6)

A certain percentage applied to one price, yields a profit that is added to that price to make a selling price.

high price = _____

percent profit = _____

low price = FINE

Variations: low price unknown (4); high price unknown (1); percent unknown (1).

A merchant sells a camera for $250. Find the cost if the profit is 25% of the cost.

Discount: Simple Discount (n = 7)

The amount of discount on an item is determined by applying the discount rate to the original price.

original price = _____

percent discount = _____

amount of discount = FIND

Variations: amount of discount unknown (5); percent unknown (1); original price unknown (1).

Tyrone gets a 10% discount at the Stereo Center. He got $4 off on a tape recorder. What was the regular price?

Table 6 (continued)

Discount: Simple Cost (n = 8)

The price of an item is discounted by a certain percentage.

discount price = _____

percent discount = _____

original price = FIND

Variations: original price unknown (5); percent unknown (2); discount price unknown (1).

An appliance store drops the price of a certain type of TV 18% to a sale price of $410. What was the former price?

Rate: Simple Rate (n = 8)

A certain rate applied to a certain number yields another number.

number of A units = _____

number of B units = _____

rate of A to B = FIND

Variations: rate unknown (5); number of A units unknown (2); number of B-units unknown (1).

A student ate 4 hamburgers in 16 minutes. What is the rate in hamburgers per minute?

Percent: Simple Percent (n = 26)

A certain percentage of a total yields a part.

percentage = _____

number of total = _____

number of part = FIND

Variations: part unknown (12); percent unknown (12); total unknown (2).

Suppose 10% of 1200 students were absent. How many students were absent?

Table 6 (continued)

Proportion: Simple Proportion (n = 11)

A certain proportion of a total yields a part.

proportion = _____

number in total = _____

number in part = _____

Variations: part unknown (6); total unknown (5).

Lee's batting average is .675. how many hits should Lee score in 5000 times at bat?

Fraction: Simple Fraction (n = 16)

A certain fraction of a total yields a part.

fraction = _____

number in part = _____

number in total = FIND

Variations: part unknown (6); total unknown (11).

About $2 billion is spent on advertising in magazines. Of this 4/5 is spent on T.V. ads. How much is spent on T.V. ads?

Index: Simple Index (n = 12)

A certain index applied to a part yields a total.

index = _____

number in total = _____

number in part = FIND

Variations: part unknown (8); total unknown (4).

A T.V. set with tubes uses about 640 kilowatt hours of electricity per year. This is 1.6 times that used by solid state sets. How many kilowatt hours does the solid state model use in a year?

Table 6 (continued)

Ratio: Simple Ratio (n = 17)

A number of elements is related to another number based on some ratio.

ratio of A to B = _____

number of A = _____

number of B = FIND

Variations: part unknown (13); ratio unknown (4).

The ratio of women to men in a class is 8 to 5. If there are 40 women, how many men are there?

Direct Varation: Miscellaneous (n = 36)

If a certain amount corresponds to certain number of units, then a different amount will correspond to a different number of units.

amount for A = _____

units for A = _____

units for B = _____

amount for B = FIND

Situations: shadows of varying lengths, work, physics, recipes, etc.

If a machine can make 1000 bolts in 2 hours, working at the same rate how many can it make in 5 hours?

Direct Variation: Unit-Cost (n = 19)

If a certain number of units cost a certain total amount, then a different number of units cost a different total amount.

total cost for A = _____

number of units for A = _____

number of units for B = _____

total cost for B = FIND

Table 6 continued)

Twelve slices of pizza cost $6. How much would eight slices cost?

Direct Variation: Traveling (n = 10)

If a certain distance can be covered in a certain number of days (or using a certain amount of gas), then a different distance can be covered in a different number of days (or using a different amount of gas).

distance for A = _____

number of time units for A = _____

number of time units for B = _____

distance for B = FIND

Maria traveled 700 kilometers in 5 days. At this rate how far would she travel in 24 days?

Direct Variation: Map Scale (n = 10)

If a certain length on a map corresponds to a certain actual distance, then a different length on the map corresponds to another distance.

length on map for A = _____

actual distance for A = _____

length on map for B = _____

actual distance for B = FIND

If 5 cm on a map represent 400 kilometers, what distance does 16 cm represent?

Direct Variation: Weight (n = 8)

If a certain length of material has a certain weight, then another length of the material will have a different weight.

length for A = _____

weight for A = _____

length for B = _____

Table 6 (continued)

weight for B = FIND

If 30 meters of wire weigh 8 kilograms, what will 40 meters of the same kind of wire weigh?

Inverse Variation: Pressure-Volume (n = 10)

The volume of gas under a certain pressure changes to a different volume under a different pressure.

pressure for situation A = ____

volume for situation A = ____

pressure for situation B = ____

volume for situation B = FIND

The volume of gas varies inversely as the pressure upon it. The volume of a gas is 200 cm^2 under pressure of 32 kg/cm^2. What will be its volume under a pressure of 40 kg/cm^2?

Inverse Variation: Physics (n = 7)

A rate that produces a certain amount is changed to a different rate that produces a different amount.

rate for A = ____

amount for A = ____

rate for B = ____

amount for B = FIND

Situations: Ohm's Law, Inverse Square Law for Mass, wavelength, pulley.

The current in an electrical conductor varies inversely as the resistance of the conductor. The current is 2 amps when the resistance is 960 ohms. What is the current when the resistance is 540 ohms?

Table 6 (continued)

Inverse Variation: Motion (n = 7)

If it takes a certain amount of time to travel at a certain rate, how long will it take to cover the same distance at a different rate?

rate for A = ____

time for A = ____

rate for B = ____

time for B = FIND

The time to drive a certain distance varies inversely according to the speed of the vehicle. Mary Bronson drives 47 mph for 4 hours. How long would it take her to make the same trip at 55 mph?

Inverse Variation: Work (n = 6)

If it takes some number of workers a certain amount of time to do a job, how long would it take a different number of workers?

rate for A = ____

amount for A = ____

amount for B = ____

rate for B = FIND

The time to complete a job varies inversely with the number of workers. If it takes 4 hours for 9 cooks to prepare a school lunch, how long would it take 8 cooks to prepare the lunch?

Inverse Variation: Unit-Cost (n = 3)

If a certain number of people to share the total cost, how much will the individual cost be for a different number of people?

cost per person for situation A = ____

number of people in situation A = ____

number of people in situation B = ____

Table 6 (continued)

cost per person in situation B = FIND

The cost of renting a beach cottage varies inversely as the number of people who rent the cottage. It costs $12 per person for 4 people to rent the cottage for a day. How much does it cost per person for 6 people to rent the cottage?

Wet Mixture: Add-to-Given (n = 42)

Given a certain amount of one solution, some amount of another solution is added to yield a total amount of a new solution.

percentage for solution A = _____

amount of solution A = _____

percentage for solution B = _____

percentage for total solution = _____

amount of solution B = FIND

Modifications: solution B is evaporated from total mixture (2).

A chemist has 3 L of a 5% acid solution. How many liters of a 20% solution must be added to make a mixture which is 10% acid?

Wet Mixture: Two-Absolute-Amounts (n = 18)

Some amount of one solution is mixed with some amount of a second solution to yield a total amount of a new solution.

percentage for solution A = _____

percentage for solution B = _____

percentage for total solution= = _____

amount for total solution = _____

amount of solution A (and/or amount of B) = FIND

Variations: Amount of solution A (and/or B) unknown (16); amount of final solution (2).

Table 6 (continued)

Modification: solution B is drained from total amount (2).

Dried apricots are 5% protein and dried prunes are 2% protein. How much of each type of fruit should be used to make a 100-gram mixture that is 3% protein?

Part: Number

One number is 8 more than another. Their sum is 54. Find the number.

(This is not a story problem.)

Part: Two-Pieces (n = 41)

A certain object (or amount) is broken into two parts.

total amount = ____

amount for part A = REL amount for part B

amount for part A (and/or amount for part B) = FIND

Modifications: story line for boards, ropes, wires, cables, coins, land, people, time, costs, angles, distance, tickets.

A 8-meter rope is cut into two pieces. One piece is 3 meters longer than the other. How long are the pieces?

Part: Three-Pieces (n = 7)

A certain object (or amount) is broken into three parts.

total amount = ____

amount for part A = REL amount for part B

amount for part B = REL amount for part C

amount for part A (and/or part B, and/or part C) = FIND

Modifications: story line for wires, boards, ropes, coins, angles, weights.

A 480 m wire is cut into three pieces. the second piece is three times as long as the first. The third piece is four times as long as the second. How long is each piece?

Table 6 (continued)

Age: Number

Four times Pete's age is the same as twice his age plus 34. How old is Pete?

(This is not a story problem.)

Age: Arithmetic

In 9 years Eric will be 25 years old. How old is he now?

(This is not a story problem.)

Age: Relative-Now-Then (n = 28)

Relative ages for two people now are compared to their relative ages at some other time.

age for A at time 1 = REL age for B at time 1

age for A at time 2 = REL age for B at time 2

time between time 1 and time 2 = ____

age for A at time 1 (and/or age for B at time 1) = FIND

Modifications: use different time 2 for A then for B (2); relation is sum of ages (7).

Ann Teak is twice as old as her son. Ten years ago Ann was three times as old as her son. What are their present ages?

Age: Absolute-Now-Then (n = 10)

Absolute ages for two people now are compared to their relative ages at some other time.

age for A at time 1 = ____

age for B at time 1 = ____

age for A at time 2 = REL age for B at time 2

time between time 1 and time 2 = FIND

Table 6 (continued)

A man is now 40 years old and his son is 14 years old. A number of years from now the father will be twice as old as his son. What is this number of years?

Consecutive Integer: Number

The sum of three consecutive odd integers is 189. What are the integers?

(This is not a story problem.)

Consecutive Integer: Sum (n = 4)

The sum of several consecutive integers is given.

number of integers = _____

sum of integers = _____

value of each integer = FIND

The five members of a cross country team finished in consecutive order. The team score was 45. Find the place number of each runner.

Rectangle: Non-Story

The length of a rectangle is 2 inches greater than the width and the perimeter is 36 inches. What is the length and width of the rectangle?

(This is not a story problem.)

Rectangle: Simple Area (n = 10)

The area of a rectangle can be determined by multiplying length times width.

length = _____

width = _____

area = FIND

Variations: area unknown (5), width unknown (3), length unknown (2).

The Kroger's rectangular garden measures 12 yards by 15 yards. What is the area of their garden?

Table 6 (continued)

Rectangle: Frame 1 (n = 11)

A frame with a certain width surrounds a rectangle.

length of large rectangle = _____

width of large rectangle = _____

area of small rectangle = _____

width of frame = FIND

A framed mirror is 40 cm by 55 cm. 1924 cm^2 of the mirrors shows. Find the width of the frame.

Rectangle: Area-Relative (n = 10)

A certain area of a rectangle occurs when the length is related to width in a certain way.

length = REL width

area = _____

length (and/or width) = FIND

The length of a rectangular window pane is twice its width. The area of the pane is 98 cm^2. What are the dimensions of the pane?

Rectangle: Perimeter (n = 8)

A certain perimeter of a rectangle occurs when the length is related to the width in a certain way.

length = REL width

perimeter = _____

length (and/or width) = FIND

Modifications: value for half the perimeter is given (2).

A rectangular palyground is 60 meters longer than it is wide. It can be enclosed by 920 meters of fencing. Find its length.

Table 6 (continued)

Rectangle: Absolute Frame 2 (n = 5)

A frame with a certain width surrounds a rectangle.

length of small rectangle = _____

width of small rectangle = _____

area of frame = _____

width of frame = FIND

Modification: area of frame is same as area of small rectangle (3)

Mr. Serena wants to double the area of his garden by adding a strip of uniform width along each of the four sides. The original garden is 12 ft by 18 ft. How wide a strip must be added?

Rectangle: Frame-Relative 1 (n = 1)

A frame with a certain width surrounds a rectangle.

length of small rectangle = REL width of small rectangle

width of frame = _____

area of frame = _____

length of large rectangle (and/or width) = FIND

The length of Hillcrest Park is 6 feet more than its width. A walkway 3 feet wide surrounds the outside of the park. The total area of the walkway is 288 square feet. Find the dimensions of the park.

Rectangle: Frame-Relative-2 (n = 1)

A frame with a certain width surrounds a rectangle

width of frame = _____

area of small rectangle = REL area of large rectangle

length (and/or width) of large rectangle = FIND

Table 6 (continued)

A picture has a square frame that is 5 cm wide. The area of the picture is two-thirds of the total area of the picture and the frame. What are the dimensions of the frame?

Circle: Word

A circle has a radium 35 cm. Find the circumference.

(This is not a story problem.)

Circle: Simple Circumference (n = 7)

Given the circumference, find the radius; given the radius find the circumference.

circumference = _____

radius = FIND

Variations: radius unknown (5), circumference unknown (2).

The circumference of a clock face is 880 cm. The minute hand touches the outside of the clock face. How long is the minute hand?

Triangle: No Story

For a right triangle, what is the length of the hypotenuse if the two other sides are 3 and 4 inches?

(This is a not a story problem.)

Triangle: Simple Pathagorean (n = 16)

Given the length of two sides of a right triangle, find the length of the remaining side.

length of side a = _____

length of side b = _____

length of hypotenuse = FIND

Variations: hypotenuse known (10), side a or b unknown (6).

Table 6 (continued)

A 26-foot ladder is leaning against a building. The foot of the ladder is 10 feet away from the building. How far above the ground does the ladder touch the wall?

Fulcrum: Two-Weights-Absolute (n = 14)

One object is positioned a certain distance from a fulcrum such that it is balanced with another object that is positioned a certain distance on the other side.

weight of A = _____

distance of A from fulcrum = _____

weight of B = _____

distance of B from fulcrum = FIND

Variations: distance unknown (6); weight unknown (8).

Laurie weighs 60 kg and is sitting 165 cm from the fulcrum of a seesaw. Bill weighs 55 kg. How *far* from the fulcrum must Bill sit to balance the seesaw?

Fulcrum: Two-Weights-Relative (n = 2)

One object is positioned a certain distance from a fulcrum such that it is balanced with another object that is positioned a certain distance on the other side.

weight of A = _____

weight of B = _____

distance A = REL distance B

distance A (and/or B) = FIND

Variations: Distance unknown (1); weight unknown (1).

Tina and Wilt are sitting 4 meters apart on a seesaw. Tina weighs 65 kg, and Wilt weighs 80 kg. How far from the fulcrum must Tina be sitting if the seesaw is in balance?

Table 6 (continued)

Fulcrum: Three-Weights (n = 1)

Two objects are one side of a fulcrum and are balanced by one object on the other side.

weight of A = ____

weight of B = ____

weight of C = ____

distance for C = ____

distance for A = REL distance for B

distance for A (and/or distance for B) = FIND

Bud and Bob balanced Neal on a seesaw. Bud weights 44 kg and sits 1 meter farther from the fulcrum than Bob, who weights 40 kg. Neal weighs 70 kg and balances Bud and Bob by sitting 3 meters from the fulcrum. How far from the fulcrum do Bud and Bob sit?

Exponential: Decay (n = 10)

An amount decays at certain rate.

decay rate = ____

initial amount = ____

ending amount = ____

time = FIND

Variations: time unknown (6), amount unknown (4).

A certain element has a half life of 2 years. If there is 60 pounds of the material, how much time will it take until there is less than one pound?

Exponential: Rebound-1 (n = 7)

A ball bounces less high on each successive bounce.

starting value ____

Table 6 (continued)

percent dampening = _____

total distance = FIND

A silicon ball dropped 12 feet rebounds, 7/10 of the hight from which it fell. How far will it travel before coming to rest?

Exponential: Rebound 2 (n = 5)

A ball bounces less high on each successive bounce.

starting value = _____

percent dampening = _____

distance on certain bounce = FIND

A golf ball dropped from the height of 81 meters rebounds on each bounce 2/3 of the distance from which it fell. How far does it fall on its 6th descent?

Series: Increment-Decrement (n = 8)

A certain number is incremented by a constant and added to total, recrusively.

amount of increment = _____

number of increments = _____

starting value = _____

ending value = FIND

Modifications: decrement from number to one (2).

Joan saves 1 dime the 1st day, 2 dimes the 2nd day, 3 dimes the 3rd day, and so on. How much money will she have saved at the end of 30 days?

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